PHYSICAL REVIEW E VOLUME 59, NUMBER 6 JUNE 1999

Stress in conic piles determined by a centrifuge experiment: Breakdown of scaling hypothesis

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(Received 6 October 1998)

It is found experimentally that vertical-stress field in a conic pile depends on gravity level, building process, and loading story. For instance, a conic pile with inclined strata does exhibit a minimum of stress in the center, whereas conic pile with horizontal strata does not; both piles exhibit an arching effect, which increases with gravity. This questions the assumptions of radius stress field scaling. Amplitude of the stress dip is found to be 10%, which is much smaller than what was found in previous experiments. [S1063-651X(99)50706-X]

PACS number(s): 83.70.Fn, 45.05.+x, 05.40.−a

Recently, a series of works [1–7] has been devoted to the stress distribution below a conic sand pile built at its angle of repose by letting sand flow from a fixed hole located just above the pile center. It has been found in particular, that the vertical-stress distribution exhibits a minimum at the pile center [1,6]. A theoretical explanation of this minimum has been proposed recently [2,3], which states the existence of some relationships between stresses, which is engraved by the building process. Similar approaches were already initiated in older mechanics works [8], but their efficiency was discussed in the framework of classical elastoplastic modeling [4,5]. In particular, Ref. [5] also finds the series of stress field found in Ref. [2].

In addition, the theoretical approach [2,3] uses also a scaling argument called radius stress field (RSF) scaling as a prime hypothesis; thus, it assumes a peculiar form of the stress field, which implies in turn that change of gravity G does not affect the stress field. The validity of this hypothesis has been already queried in Ref. [7] in view of results on slope stability and avalanches [9], but it has never been checked experimentally directly. This is done here by using a centrifuge [10]; the experiments also test the effect of a change of the building process on the stress distribution. Under the experimental conditions, it is found (i) that the vertical stress increases with G in the wing of the pile but decreases in the center so that stress distribution does depend on gravity, contrary to the scaling hypothesis; (ii) that stress distribution depends on the building process since the conic pile built with inclined strata does exhibit a stress minimum at the center, but not the piles built with horizontal strata; (iii) that stress does evolve during the first loading-unloading cycles till a stationary (i.e., ‘‘engraved’’) regime is reached. So, points (i) and (iii) demonstrate that the constitutive stress relation (if it exists) and/or the boundary conditions depend on (a) the number of gravity cycles performed and (b) the gravity level even when stationary regime is reached.

Two different conic piles, one with horizontal strata, the other one with inclined strata, have been built on the same circular support using the pluviation method [11] from a mobile slit and a fixed hole, respectively. They are made from the same Fontainebleau sand (friction angle $\varphi=33\pm1°$, maximum density $\rho_{max}=1.739$ g/cm^3, minimum density $\rho_{min}=1.422$ g/cm^3, grain diameter $d=0.2\pm0.03$ mm). This support, whose diameter is 604 mm, contains eight flat vertical-stress gauges of diameter $D=75$ mm [11–13] located at different distances R from the center ($R=20$, 70, 100, 130, 160, 190, 220, and 250 mm) and in two directions (Ox,Oy) perpendicular to each other. Positions of each gauge remained unchanged for the two piles. The support with the pile is placed in the basket of the Laboratoire Central des Ponts et Chausseés (LCPC) centrifuge [10], which is run at different values of the efficient ‘‘gravity’’ G; G will be given in units of Earth gravity $g=9.81$ m/s^2. Obviously, the increase of G needs a transverse horizontal acceleration $\gamma=R\,d\omega/dt$ (i.e., perpendicular to the axis of rotation and to the radius of the centrifuge). γ is small, though (i.e., <0.3 m/s^2=g/30), but it is sufficient to generate avalanches during the first run only, since the pile is built just at the limit of equilibrium; however, the pile geometry is only slightly perturbed by this acceleration since its summit moves only 1 cm, which corresponds to other imperfections of geometry linked to dilatancy and/or cohesion effect.

Figure 1 shows the applied gravity as a function of time. The first loading-unloading cycle at 2 g is needed to get a stable configuration of pile as mentioned above; (no avalanching and change of shape occurs after the first run); then six different G levels are achieved in the following order: 50, 15, 15, 15, 50, and 50 g; in between each plateau the gravity is reduced to 1 g. This series of gravity allows to determine

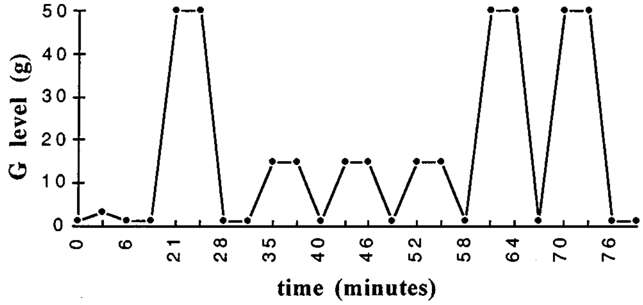

FIG. 1. Applied gravity (in g=9.81 m/s^2 units) vs time for both conic piles of Fontainebleau sand (with inclined and horizontal strata).

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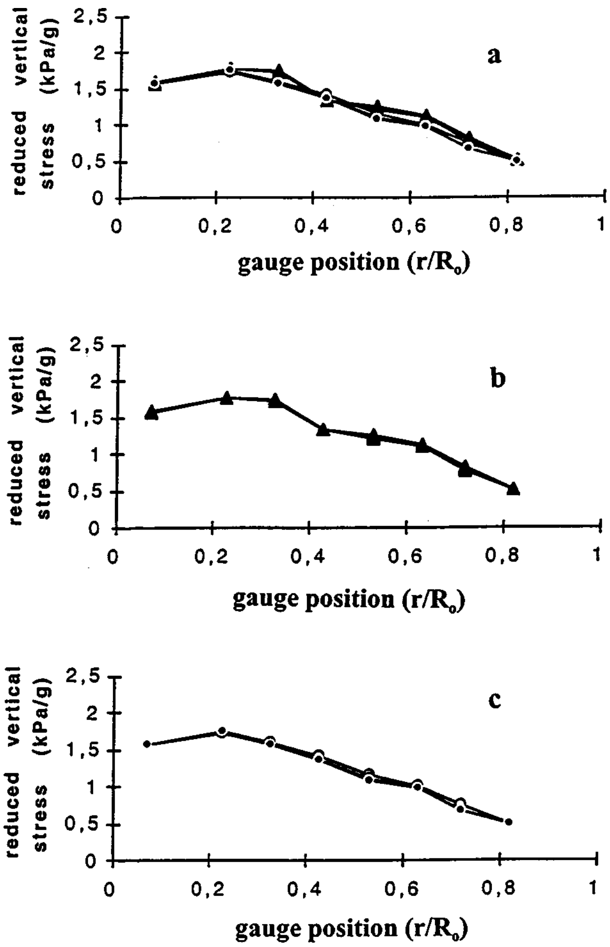

FIG. 2. Vertical-stress distribution at 15 and 50 g under a conic pile with inclined strata in reduced scales σ/G, r/R_0 (R_0=radius of the cone basis). White symbols correspond to first loading at 50 g (circles) and at 15 g (triangles). Triangles, 15 g; squares, lozenges, and circles, 50 g. (a) 15 and 50 g experiments: all points are localized around a single curve, which exhibits a minimum at pile center. (b) Three different 15 g experiments: stress distribution is quite stable (within 1%); one can observe a tiny consolidation effect between the first and second loadings. (c) Three different 50 g experiments; one can observe a consolidation effect between the first and second loadings.

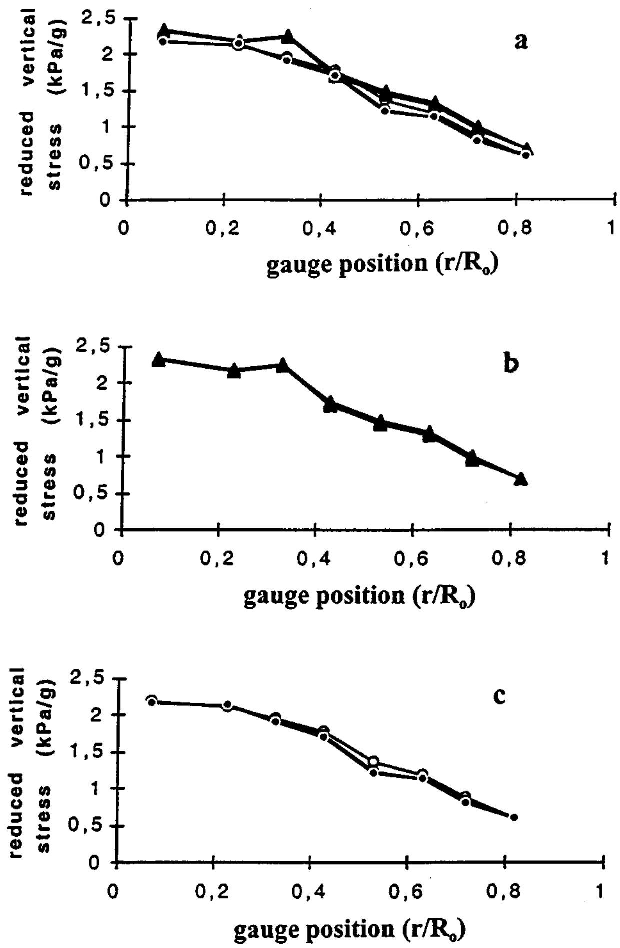

FIG. 3. Vertical-stress distribution at 15 and 50 g under a conic pile with horizontal strata in reduced scales σ/G, r/R_0 (R_0=radius of the cone basis). White symbols correspond to first loading at 50 g (circles) and at 15 g (triangles). (a) 15 and 50 g experiments: all points are localized around a single curve. (b) Three different 15 g experiments: stress distribution is quite stable (within 1%); one an observe a tiny consolidation effect between the first and second loadings. (c) Three different 50 g experiments; one can observe a consolidation effect between the first and second loadings.

some history dependence due to cycling and evolution of G. Furthermore, each plateau lasts about 3 min, during which the stress distribution and gravity are measured a few times; this allows us to determine the stress fluctuations; it is found to be less than 0.3% at 50 g and 0.5% at 15 g. We will take this value as the intrinsic noise of the gauge. Another source of noise comes from the number of grains in contact with the gauge; as a matter of fact, it is accepted that contact forces between grains are random variables whose fluctuation amplitude is equal to the mean force [14] so that stress fluctuations $\delta\sigma$ decreases as the square root of the number of grains in contact with the gauge; one expects then that the stress field cannot be determined with better accuracy than $\delta\sigma = \sigma d/D = 0.3\%$ in the present case; this is consistent with experimental data.

Figures 2 and 3 report the reduced vertical-stress distribution σ/G measured by the eight gauges at the different locations for the different G levels, in reduced units. When plotted all together [Figs. 2(a) and 3(a)], the data seem to fall into a single curve with a dispersion less than 10%. But, comparing Figs. 2(a) and 3(a) one notes that this single curve depends on the way the pile has been built: it exhibits a minimum at the pile center when the pile is built with inclined strata, but does not exhibit this minimum when strata are horizontal. However, dispersion of data point ($=10\%$) is much larger than the experimental error given in the previous paragraph ($=0.3\%$), so that it can reveal a systematic fluctuation of the stress field at a given G when G is decreased and increased again, or simpler, a dependence of the stress field distribution upon G.

Figures 2(b), 2(c), 3(b), and 3(c), which report the result of the experiments at 15 and 50 g separately for the two different piles, demonstrate that the dispersion is much less when a single value of G is concerned; for instance, it is about 1% for the 15 g experiments and for the two last 50 g experiments. These figures demonstrate that the real stress depends on G. One can also observe a slight consolidation effect in Figs. 2(c) and 3(c), since at 50 g, the stress field of the first loading is different from the two others; a similar consolidation effect can be observed at 15 g [Figs. 2(b) and 3(b)] but the observed variations are much less. This last point is not surprising, since the piles have been loaded to 50 g already prior to these experiments (cf. Fig. 1); this limits the amplitude of the consolidation effect.

So, as Figs. 2 and 3 demonstrate, stress distribution *(σ/G)*, expressed in reduced units, depends on *G* in the present experiment. This phenomenon is observed for both piles; this demonstrates that stress field distribution does depend on *G* and that RSF scaling is not satisfied in the present experiment.

It is also worth noting that the stress dip, which is found in the case of the conic pile with inclined strata, has an amplitude (10%) smaller than in the case of Ref. [1] (50%). Such a difference between these two amplitudes may be explained either by some binding of the support which can be larger in the previous case than in our experimental one [4], or by the stiffness of the gauge, which is larger in these experiments than in Smid and Novosad ones, or even by some spontaneous density increase of the sand during the building process, which can be larger in the case of Ref. [1] than in our own experiment: recent computer simulations [15] using nonlinear elastoplastic modeling [16] with two plastic mechanisms predicts such a large dip in a loose pile: it predicts that sand density increases with stress, which reports the weight of the central part on the wings enhancing the dip. However, as a small dip can be observed only in the present case, we do believe that the fall height (80 cm) from which sand has been dropped during pile building, is large enough to generate a pile dense enough and to cancel this effect. This height is probably larger than in the case of Ref. [1].

It is also possible to compare these results with computation using the three following hypotheses: (i) a simple elastic-perfectly plastic modeling, (ii) rigid boundary conditions, and (iii) different building processes (inclined strata and horizontal strata), for which we have obtained preliminary results [15]. This computation finds a dip of 10% in the center of the pile for the conic pile built with inclined strata; it finds no dip for the conic pile with horizontal strata, and no dip for the 2D triangular pile whatever the building process. Furthermore, these computations predict a stress at the center which is 2.15 kPa/g for the conic pile of Fontainebleau sand built with horizontal strata at a density $\rho=1.585$ kg/m^3 (which is this one generated by the pluviation characteristics); it predicts also a stress at dip σ/g=1.56 kPa/g (1.64 kPa/g) and a stress at maximum σ/g=1.72 kPa/g (1.82 kPa/g) when the conic pile of Fontainebleau sand with inclined strata has a density $\rho=1500$ kg/m^3 (1.585 kg/m^3). So, these simulations compare well with our experimental data 2.2 kPa/g for the conic pile with horizontal strata and 1.55 kPa/g (and 1.75 kPa/g) for the dip (and maximum) in the case of the conic pile with inclined strata [15].

Nevertheless the differences between experimental data of 15 and 50 g are not described by the elastic-modeling with simply rigid boundary conditions. This would require at least to make use moving-boundary conditions. One can remark that distributions at 50 g are smoother than at 15 g in Figs. 2 and 3, which might be due to such an evolution of boundary condition and/or might reveal some inhomogeneity of the pile. On the other hand, are these differences able to query the engravement of a constitutive relation [2]? Probably not, since we do not have enough data and relations.

This paper shows that more investigation has to be performed for a complete understanding of the problem. It is also aimed at emphasizing the difficulty of getting an exact rigorous solution to a ''simple'' experiment. Furthermore, we shall recall that the reconstruction of a stress field from a few data points is always a difficult operation that requires important assumptions; this is due to the fact that the inverse problem does not have a unique solution.

Anyway, the RSF scaling [2] is probably not valid; this is not bothersome really, since the explanation of the dip does not rely on it: first we notice that the Wittmer *et al.* approach demonstrates the existence of a series of possible solutions exhibiting a dip in the center and obeying a high degree of symmetry. Second we note (i) that reducing the symmetry of the problem increases the number of possible solutions, and (ii) that the solutions with higher symmetries fall as peculiar cases of this new series. So, we conclude that a few adequate stress field solutions shall also exist exhibiting a lower symmetry and a dip of vertical stress in the center.

On the contrary, let us assume for a while that the RSF scaling and Wittmer model are really valid in pile experiments [1]. The right question would be in this case to understand why, since it is a ''fragile'' matter according to this approach [3].

Experiments have been funded by the LCPC. P.E. wants to thank J.-P. Bouchaud and J. Garnier for interesting discussions, O. Ternet for his technical help, and Centre National d'Etudes Spatiales for partial funding.

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[10] The LCPC centrifuge, which has been used is the model 680 from Acutronics with a 5.5 m radius, which allows one to apply 2000 m/s^2 to 500 kg.

[11] Description of pluviation (or raining) technique can be found in note 11 of P. Evesque *et al.*, Phys. Rev. E **47**, 2326 (1993). The gauges have been designed by Askegaard (Ref. [12]) to measure stress in granular material. They consist of a thin flat cylindrical chamber made of titanium filled with oil whose pressure is measured. Their external diameter is 75 mm, the inner height of the chambers is approximately 0.1 mm, the total height of the gauges is 14 mm, which includes the height of the pressure transducer. The gauge stiffness is ensured by the small compressibility of oil; its maximum deformation is less than 1 μm for 100 kPa as recommended [17b]; it is three times better than in Ref. [11]. More information about the gauges can be found in Refs. [12] and [13] and on the difficulty of measuring stress with other more classical gauges in Ref. [17].

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